MATHEMATICS FOR POLITICAL SCIENCE

(PRELIMINARY SYLLABUS: NOT FINALIZED & MAY CONTAIN SMALLER MISTAKES)

Winter Semester 2021–2022, University of Konstanz

Seminar Time:	Tuesday, 5.00–6.30 PM (17.00–18.30 Uhr)
Seminar Location:	D436
Instructor:	Jan P. Vogler
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Tutorial Time:	TBD
Tutorial Location:	TBD
Teaching Assistant:	Marius Kaltenbach
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Course Abstract:

The application of rigorous statistical methods is a core aspect of modern political research. Moreover, many key contributions to political science and political economy are based on game theoretic modeling. In order to fully understand these statistical and game theoretic approaches, comprehensive knowledge of the underlying mathematical tools is essential. Therefore, this class introduces students to a number of topics in mathematics that are a prerequisite to advanced classes in methodology: (1) We begin with a quick introduction to the fundamentals of mathematics, including mathematical notation, functions, limits, and other basic topics. (2) Then we study calculus in one dimension, including differentiation, integration, and the identification of extrema. (3) Probability theory is an essential building block of mathematical statistics, which is the reason for us to devote a significant amount of time to this topic. (4) The fourth topic is linear algebra, including systems of equations, Eigenvalues, and Markov Chains. (5) Finally, the class closes with an introduction to multivariate calculus. Knowledge of all of these tools will enable the students to subsequently take more advanced methodological classes in statistics and game theory.

Course Objectives:

By the end of the class, students will be able to:

- Understand the fundamental building blocks of mathematics, including mathematical notation, functions, sequences and series, and more.

- Describe the rules of calculus in one dimension with respect to differentiation, integration, and the evaluation of extrema.
- Elaborate on key components of probability theory, which entail different types of statistical distributions and probability functions, among others.
- Apply the tools of linear algebra to vectors and matrices, solve systems of equations, and find Eigenvalues of matrices.
- Combine several of the mathematical tools we have learned about to identify extrema in multivariate functions.

Course Requirements:

Useful Background Knowledge

Basic knowledge of mathematics, up to calculus in one dimension, will be very useful for this class. While students without prior training in calculus are equally encouraged to enroll in the course, the class might be slightly more challenging for them. In general, the course is designed in a way that anyone—even students with very little prior mathematics training or for whom such training is in the distant past—have a chance to succeed in this class. Throughout the semester, I will be available to give further advice and guidance to students who want to catch up on any the aforementioned subjects.

Preparation for Class, Weekly Readings, and Participation

Students are asked to study the textbook along the class. While students are encouraged to read the relevant chapters prior to class to familiarize themselves with terms, notation, and content, they may choose to first come to class and consult the textbook afterwards.

Written Assignments

There are two types of written assignments in this class. Specifically, students are expected to submit four problem sets and participate in one final examination.

Requirements for the problem sets (50% of the course grade): Four times throughout the semester, students will receive a problem set. Problem sets will typically include 5-10 individual tasks that are meant to test students' understanding of the past sessions of the class. These problem sets are meant to consist of a mix of easy, medium, and difficult tasks. Students are welcome to discuss possible strategies to solve problems with their peers, but they are required to never copy any solution directly from someone else and they are required to write down the submitted solution completely independently. The specific dates for the problem sets can be found in the course schedule below.

Requirements for the final examination (50% of the course grade): The final examination will cover the entire content of the class. It is meant to provide students with an opportunity to demonstrate that they have mastered every subject that they have learned about. The final examination will take 90 minutes. The specific time of the final exam is TBD.

Late Assignment Policy

If you cannot finish an assignment on time for a legitimate reason, including, for example, emergencies and illness, please contact me. Please do so ideally before the assignment is due, but at most seven days after the deadline. In case of illness, a statement by your doctor is needed. Depending on the situation, I will provide you with an alternative assignment and/or deadline. If no legitimate reason is provided for late assignments, 10% of the point total for the assignment are subtracted for every day that the assignment is late, and a point total of 0 is awarded if the assignment is more than seven days late.

Grading:

Based on the above requirements, the course grade will consist of the following elements:

- 50%: Four Problem Sets (12.5% each)
- 50%: Final Examination

Inclusion:

An essential goal of the class is to create an open and welcoming discussion atmosphere. Diversity of opinions, constructive discussion, and mutual respect are at the core of academic discourse and will be key elements of this class. A heterogeneity in backgrounds, experiences, and identities will greatly benefit us by allowing us to learn from each other and expand our thinking. All students are encouraged to voice their opinions and to do so in a way that displays respect for the opinions of other students in the class. Students who believe that these goals are inhibited in any way should contact me so that we can discuss their concerns.

Academic Integrity:

A second essential goal of mine is to uphold the standards of academic integrity in this class. It is expected that all work submitted is entirely done by the person who submits it. All literature used in writing a paper must be referenced. Students are expected to always use quotation marks when they directly quote the words or statements of others. We will discuss best practices for citation during the first session of class. Students are also expected to turn in a full bibliography listing works consulted during their research for each written assignment. If you have any questions about academic integrity, please contact me so that we can discuss them.

Books for Purchase:

The class is primarily based on the following book. All students are required to acquire it:

- Moore, W. H., & Siegel, D. A. (2013). *A mathematics course for political and social research*. Princeton University Press. **[Moore & Siegel]**

Important Dates and Deadlines:

- First class: Tuesday, October 26, 2021
- Holiday break day I (no class): Tuesday, December 28, 2021
- Holiday break day II (no class): Tuesday, January 4, 2021
- First problem set due: Tuesday, November 16, 2021
- Second problem set due: Tuesday, December 7, 2021
- Third problem set due: Tuesday, January 11, 2022
- Fourth problem set due: Tuesday, February 1, 2022
- Last class: Tuesday, February 8, 2022
- Final examination: TBD

Tutorial:

Once per week, there will be a 90-minute tutorial led by the teaching assistant. Participation in the tutorial is a regular aspect of the class and thus expected, except for cases of illness or emergency. The tutorial will be organized in the following way: For 30 minutes, the teaching assistant will review key concepts from that week's class. If any material was missed in class, the teaching assistant may cover (parts of) it in the tutorial. For another 30 minutes, the teaching assistant will go through prepared exercises with the students. For the last 30 minutes, students can ask questions (e.g., "Can you explain this concept again?", "How do I solve this problem?"). If there is anything specific that you would like the teaching assistant to cover in the tutorial, please email the teaching assistant at least 6 hours before the beginning of the tutorial so that he/she/they can prepare.

Office Hours:

The time and location of the office hours are TBD.

Course Schedule Begins on the Following Page.

COURSE SCHEDULE:

PART I: INTRODUCTION TO THE CLASS AND TO MATHEMATICS FOR POLITICAL SCIENCE

The first part of the class introduces the students to the class schedule and to the fundamentals of mathematics. In the first session, we discuss why we need to study mathematics if we want to do cutting-edge political research. Moreover, this part covers (1) variables and constants, (2) sets, (3) operators, (4) relations, (5) levels of measurement, (6) notation, (7) an introduction to proofs, (8) basic properties of arithmetic, (9) algebra review, (10) computational aids, (11) functions, (12) preference relations and utility functions, (13) sequences and series, (14) limits, (15) sets, (16) continuous functions.

1. **Introduction and Course Overview: "Mathematics for Political Science"** **(October 26, 2021)**

   The Relevance of Mathematics for Modern Research in Political Science

Required Reading:

- No specific required readings for the first class, but I will circulate some research articles that use statistical research methods to underscore the importance of mathematics to modern political research. Please pick one that is of greatest interest to you and read it more carefully.

2. **Building Blocks, Part I** **(November 2, 2021)**

   Preliminaries + Algebra Review

Required Readings:

- **Moore & Siegel,** Ch. 1 ("Preliminaries")
- **Moore & Siegel,** Ch. 2 ("Algebra Review")

3. **Building Blocks, Part II** **(November 9, 2021)**

   Functions, Relations, & Utility + Limits and Continuity, Sequences & Series, More On Sets

Required Readings:

- **Moore & Siegel,** Ch. 3 ("Functions, Relations, and Utility")
- **Moore & Siegel,** Ch. 4 ("Limits and Continuity, Sequences and Series, and More On Sets")

PART II: CALCULUS IN ONE DIMENSION

The second part of the class deals with calculus in one dimension. Specifically, this part covers (1) introduction to calculus, (2) the derivative, (3) rules for differentiation, (4) derivatives of functions, (5) the definite integral, (6) indefinite integrals and the fundamental theorem of calculus, (7) computing integrals, (8) rules of integration, (9) extrema, (10) higher-order derivatives, concavity, and convexity, and (11) finding extrema.

4. Calculus, Part I: Fundamentals (Differentiation) **(November 16, 2021)**

Introduction to Calculus and the Derivative + The Rules of Differentiation

Required Readings:

- **Moore & Siegel,** Ch. 5 ("Introduction to Calculus and the Derivative")
- **Moore & Siegel,** Ch. 6 ("The Rules of Differentiation")

Problem Set 1 Due at the Beginning of Class on November 16, 2021!

5. Calculus, Part II: The Integral **(November 23, 2021)**

Required Reading:

- **Moore & Siegel**, Ch. 7 ("The Integral")

6. Calculus, Part III: Extrema in One Dimension **(November 30, 2021)**

Required Reading:

- **Moore & Siegel**, Ch. 8 ("Extrema in One Dimension")

PART III: PROBABILITY

The third part of the class deals with probability theory. Specifically, it covers (1) basic probability theory, (2) computing probabilities, (3) specific measures of probabilities, (4) the distribution of a single variable, (5) sample distributions, (6) empirical joint and marginal distributions, (7) the probability mass function, (8) the cumulative distribution function, (9) probability distributions and statistical modeling, (10) expectations of random variables, (11) continuous random variables, (12) expectations of continuous random variables, (13) important continuous distributions for statistical modeling.

7. Probability, Part I: An Introduction to Probability **(December 7, 2021)**

Required Reading:

- **Moore & Siegel**, Ch. 9 ("An Introduction to Probability")

Problem Set 2 Due at the Beginning of Class on December 7, 2021!

8. Probability, Part II: Discrete Distributions **(December 14, 2021)**

Required Reading:

- **Moore & Siegel**, Ch. 10 ("Discrete Distributions")

9. Probability, Part III: Continuous Distributions **(December 21, 2021)**

Required Reading:

- **Moore & Siegel**, Ch. 11 ("Continuous Distributions")

– NO CLASS ON DECEMBER 28, 2021 (HOLIDAY BREAK)! –

– NO CLASS ON JANUARY 4, 2022 (HOLIDAY BREAK)! –

PART IV: LINEAR ALGEBRA

The fourth part of the class deals with linear algebra. Specifically, it covers (1) scalars and vectors, (2) matrices, (3) properties of vectors and matrices, (4) matrix illustration of OLS estimation, (5) vector spaces, (6) solving systems of equations, (7) Eigenvalues, Eigenvectors, and matrix decomposition, (8) Markov chains and stochastic processes.

10. Introduction to Linear Algebra **(January 11, 2022)**

Required Reading:

- **Moore & Siegel**, Ch. 12 ("Fun with Vectors and Matrices")

Problem Set 3 Due at the Beginning of Class on January 11, 2022!

11. Vector Spaces and Systems of Equations **(January 18, 2022)**

Required Reading:

- **Moore & Siegel**, Ch. 13 ("Vector Spaces and Systems of Equations")

12. Eigenvalues and Markov Chains **(January 25, 2022)**

Required Reading:

- **Moore & Siegel**, Ch. 14 ("Eigenvalues and Markov Chains")

PART V: MULTIVARIATE CALCULUS

The fifth and final part of the class deals with multivariate calculus. Specifically, it covers (1) functions of several variables, (2) calculus in several dimensions, (3) concavity and convexity redux, (4) unconstrained optimization, (5) constrained optimization with equality constraints, (6) constrained optimization with inequality constraints, (7) properties of the maximum and minimum, and (8) implicit differentiation.

13. Introduction to Multivariate Calculus **(February 1, 2022)**

Required Readings:

- **Moore & Siegel**, Ch. 15 ("Multivariate Calculus")
- **Moore & Siegel**, Ch. 16 ("Multivariate Optimization") [esp. pp. 376–383 on Unconstrained Optimization + Related Exercises]

Problem Set 4 Due at the Beginning of Class on February 1, 2022!

14. Further Topics in Multivariate Calculus **(February 8, 2022)**

Constrained Optimization + Comparative Statics and Implicit Differentiation

Required Readings:

- **Moore & Siegel**, Ch. 16 ("Multivariate Optimization") [esp. pp. 383–398 on Constrained Optimization + Related Exercises]
- **Moore & Siegel**, Ch. 17 ("Comparative Statics and Implicit Differentiation")

Dates and Deadlines at the End of the Semester:

- Final Examination: TBD