Mathematics and Probability Theory for Political Science Research

The final version is available at this link.

Fall Semester 2025, Aarhus University

Seminar Time:	**Monday, 10.00 AM – 1.00 PM (10.00–13.00)**
Seminar Location:	**Various (see syllabus p. 5 for details) (in-person seminar)**
Instructor:	**Jan P. Vogler (Website: janvogler.net)**
Contact:	**jan.vogler@ps.au.dk**

Course Abstract:

The application of rigorous statistical methods is a core aspect of modern political science research. Moreover, many key contributions to political science and political economy are based on game theoretic modeling. In order to fully understand these statistical and game theoretic approaches, comprehensive knowledge of the underlying mathematical tools is essential. Therefore, this class introduces Ph.D. students to a number of related topics: (1) We begin with a quick introduction to the fundamentals of mathematics, including mathematical notation, functions, limits, and other basic topics. (2) Then we study calculus in one dimension, including differentiation, integration, and the identification of extrema. (3) Probability theory is an essential building block of statistical analysis, which is the reason for us to devote a significant amount of time to this topic. (4) The fourth topic is linear algebra, including systems of equations and Eigenvalues. (5) In addition, we also discuss key aspects of multivariate calculus, especially the optimization of multivariate functions. (6) In the final part of the course, building up on all previous insights, we consider the mathematical properties of regression analysis and maximum likelihood estimation. Knowledge of these tools will enable the students not only to better understand the application of statistical methods in modern research, including methods of maximum likelihood, but also to take advanced methodological courses.

Course Objectives:

By the end of the class, students will be able to:

- Understand the fundamental building blocks of mathematics, including mathematical notation, functions, sequences and series, and more.
- Describe the rules of calculus in one dimension with respect to differentiation, integration, and the evaluation of extrema.

- Elaborate on key components of probability theory, which entail different types of statistical distributions and probability functions, among others.
- Apply the tools of linear algebra to vectors and matrices, solve systems of equations, and find Eigenvalues of matrices.
- Combine several of the mathematical tools we have learned about to identify extrema in multivariate functions.
- Use their newly acquired knowledge of mathematics and probability theory to better understand regression analysis and maximum likelihood estimation in social research.

Course Requirements:

Useful Background Knowledge

Basic knowledge of mathematics, up to calculus in one dimension, will be very useful for this class. While students without prior training in basic mathematics and calculus are equally encouraged to enroll in the course, the class might be slightly more challenging for them. In general, the course is designed in a way that anyone—even students with little prior training in mathematics or for whom such training is in the distant past—have a chance to succeed. Throughout the semester, I will be available to give further advice and guidance to students who want to catch up on any basic subject.

Preparation for Class, Weekly Readings, and Participation

Students are asked to carefully read and follow the relevant textbook chapters while they are enrolled in the class. While the course participants are encouraged to study the relevant chapters prior to class to familiarize themselves with terms, notation, and content, they may also choose to first come to class and consult the textbook afterwards, if this approach is more conducive to their learning.

Written Assignments (7 ECTS version, "Standard Version" of the Course)

In the 7 ECTS version of the course, the entire grade will be determined by a final written examination.

Requirements for the final examination (100% of the course grade): The final examination will cover parts I through V of the course (part VI will ***not*** be covered). It is meant to provide students with an opportunity to demonstrate that they have mastered the course's core subjects. The final examination will take 180 minutes. The final examination will take place on **January 23, 2026.**

Written Assignments (10 ECTS version)

In the 10 ECTS version of the course, there will be two written assignments. Specifically, students who are enrolled in this version are expected to submit four problem sets and participate in one final examination.

Requirements for the problem sets (40% of the course grade): Four times throughout the semester, the students will receive a problem set and will have approximately two weeks to complete it. Problem sets will typically include 10 individual tasks that are meant to test students' understanding of the past sessions of the class. They will consist of a mix of easy, medium, and difficult tasks. Students are welcome to discuss possible strategies to solve problems with their peers, but they are required to never copy any solution directly from someone else. Moreover, they are required to write down the submitted solution entirely independently. If I find clear evidence that answers were directly copied between two students, both students will fail the respective assignment. The specific dates for the problem sets can be found in the course schedule below.

Requirements for the final examination (60% of the course grade): The final examination will cover parts I through V of the course (part VI will ***not*** be covered). It is meant to provide students with an opportunity to demonstrate that they have mastered the course's core subjects. The final examination will take 180 minutes. The final examination will take place on **TBD**.

Late Assignment Policy

If you cannot finish an assignment (especially the problem sets) on time for a legitimate reason, including, for example, emergencies and illness, please contact me as soon as possible. In coordination with the department administration and in line with the department's examination rule, we will deal with such situations on a case-by-case basis.

Attendance Policy

Students are expected to come to as many sessions of the course as possible. In line with official departmental policy, they are required to participate in at least 80% of all sessions. Furthermore, students must notify the lecturer if they are unable to attend class. Students are also expected to actively participate in the course, for example by answering questions and participating in in-class exercises. Participation in the sessions is highly relevant to students' learning success. All other policies regarding attendance are in line with departmental standards.

Furthermore, students who miss a certain number of sessions will be required to provide a written overview/summary of those parts of the course. Further details on my expectations are available upon request.

Grading:

Course Grade Components (7 ECTS version)

Based on the above requirements, for 7 ECTS the grade will consist of the following elements:

- 100%: Final examination

Course Grade Components (10 ECTS version)

Based on the above requirements, for 10 ECTS the grade will consist of the following elements:

- 40%: Four problem sets (10% each)
- 60%: Final examination

Inclusion:

An essential goal of the class is to create an open and welcoming discussion atmosphere. Diversity of opinions, constructive discussion, and mutual respect are at the core of academic discourse and will be key elements of this class. A heterogeneity in backgrounds, experiences, and identities will greatly benefit us by allowing us to learn from each other and expand our thinking. All students are encouraged to voice their opinions and to do so in a way that displays respect for the opinions of other students in the class. Students who believe that these goals are inhibited in any way should contact me so that we can discuss their concerns.

Academic Integrity:

A second essential goal of mine is to uphold the standards of academic integrity in this class. It is expected that all work submitted is entirely done by the person who submits it. Although it is allowed to discuss strategies while working on problem sets, all final and submitted answers to these problem sets must be written down independently by the individual student who submits the work. The use of AI in answering the problem sets is prohibited and will represent a disadvantage to students with respect to their preparation for the final examination. If you have any questions about academic integrity, please contact me so that we can discuss them.

Textbooks:

The class is primarily based on selections from the following books:

- Moore, W. H., & Siegel, D. A. (2013). *A mathematics course for political and social research*. Princeton University Press. **[Moore & Siegel]** **[Primary Textbook]**
    - Companion Website: https://sites.duke.edu/daveasiegel/teaching/math-course/
    - Errata: https://sites.duke.edu/daveasiegel/files/2020/07/Errata.pdf
    - E-book available: https://ebookcentral-proquest-com.ez.statsbiblioteket.dk/lib/asb/detail.action?pq-origsite=primo&docID=1205618
- Gailmard, S. (2014). *Statistical modeling and inference for social science*. Cambridge University Press. **[Gailmard]**
    - E-book available: https://www-cambridge-org.ez.statsbiblioteket.dk/highereducation/books/statistical-modeling-and-inference-for-social-science/D773AAD79EE63616B01AFCD1B3EB112A#contents

- Cunningham, S. (2021). "Probability and Regression Review (Chapter 2)", in: *Causal Inference: The Mixtape*. Yale University Press. **[Cunningham]**
  - Companion Website: https://mixtape.scunning.com/
  - E-book available: https://ebookcentral-proquest-com.ez.statsbiblioteket.dk/lib/asb/detail.action?pq-origsite=primo&docID=6425560
- Pishro-Nik, H. (2014). *Introduction to probability, statistics, and random processes*. Kappa Research. **[Pishro-Nik]**
  - Companion Website: https://www.probabilitycourse.com/
- Pawitan, Y. (2001). *In all likelihood: statistical modelling and inference using likelihood*. Oxford University Press. **[Pawitan]**
  - E-book available: https://ebookcentral-proquest-com.ez.statsbiblioteket.dk/lib/asb/detail.action?pq-origsite=primo&docID=1132311

Weekly Class Structure:

Throughout most of the course, the general class structure will be the following:

1. First 45-minute block: We will work through key parts of the assigned readings together, usually on the board. Please make sure to bring writing material so you can take notes.
2. Second 45-minute block: We will work through key parts of the assigned readings together, usually on the board. Please make sure to bring writing material so you can take notes.
3. Third 45-minute block: We will either look at concrete examples of how the formulas/tools we have learned are reflected in practical empirical applications or work through practice problems as a preparation for the problem sets and exam.

Important Dates and Deadlines:

First class:	Monday, August 25, 2025
Special session I:	Friday, September 5, 2025, 1.00 PM (13.00)
Holiday break day I (no class):	Monday, September 8, 2025
Holiday break day II (no class):	Monday, September 15, 2025
Special session II:	Friday, September 19, 2025, 2.00 PM (14.00)
Holiday break day III (no class):	Monday, October 13, 2025
Break day IV (no class):	Monday, October 27, 2025
Special session III:	Friday, October 31, 2025, 2.00 PM (14.00)
Break day V (no class):	Monday, November 24, 2025
Special session IV:	Friday, November 28, 2025, 3.00 PM (15.00)
Holiday break day VI (no class):	Monday, December 1, 2025
Special session V:	Thursday, December 11, 2025, 2.00 PM (14.00)
Last class:	Thursday, December 11, 2025, 2.00 PM (14.00)
Final examination:	Friday, January 23, 2026, 2.00-5.00 PM (14.00-17.00)

Problem Set Deadlines (10 ECTS Version of the Course):

- First problem set due: Friday, September 19, 2025, 2.15 PM
- Second problem set due: Monday, October 20, 2025, 10.15 AM
- Third problem set due: Monday, November 10, 2025, 10.15 AM
- Fourth problem set due: Monday, November 24, 2025, 10.15 AM
- Please note that the problem set due dates may be adjusted by the instructor if students need more time to complete them.

Office Hours:

If you would like to speak with me, please contact me by email to set up an appointment. In the email, please include the specific reason why you would like to speak with me and provide me with at least three different dates at and time frames during which you are available.

Room Allocation:

Because we will use different rooms throughout the semester, below is an overview of which rooms we will use on which day.

Dates	Start time	End time	Rooms
Mon Aug 25 2025	10:00	13:00	1323-118 Undervisningslokale
Mon Sep 01 2025	10:00	13:00	1323-118 Undervisningslokale
Fri Sep 05 2025	11:00	12:00	1325-028 Undervisningslokale
Fri Sep 05 2025	13:00	16:00	1325-336 Undervisningslokale
Fri Sep 19 2025	14:00	17:00	1323-118 Undervisningslokale
Mon Sep 22 2025	10:00	13:00	1325-336 Undervisningslokale
Mon Sep 29 2025	10:00	13:00	1323-118 Undervisningslokale
Mon Oct 06 2025	10:00	13:00	1325-440 Undervisningslokale
Mon Oct 20 2025	10:00	13:00	1323-118 Undervisningslokale
Fri Oct 31 2025	14:00	17:00	1323-118 Undervisningslokale
Mon Nov 03 2025	10:00	13:00	1323-118 Undervisningslokale
Mon Nov 10 2025	10:00	13:00	1325-336 Undervisningslokale
Mon Nov 17 2025	10:00	13:00	1325-440 Undervisningslokale
Fri Nov 28 2025	15:00	18:00	1323-118 Undervisningslokale
Thu Dec 11 2025	14:00	17:00	1323-118 Undervisningslokale
Fri Jan 23 2026	14:00	17:00	1330-038 Undervisningslokale

Course Schedule Begins on the Following Page.

COURSE SCHEDULE:

PART I: INTRODUCTION TO THE COURSE AND TO MATHEMATICS AND PROBABILITY THEORY FOR POLITICAL SCIENCE

The first part of the class introduces the students to the course schedule and to the fundamentals of mathematics. In the first session, we discuss why we need to study mathematics and probability theory if we want to do cutting-edge political research. Moreover, this part of the class covers (1) variables and constants, (2) sets, (3) operators, (4) relations, (5) levels of measurement, (6) notation, (7) an introduction to proofs, (8) basic properties of arithmetic, (9) algebra review, (10) computational aids, (11) functions, (12) preference relations and utility functions, (13) sequences and series, (14) limits, (15) sets, and (16) continuous functions.

1. **Introduction and Course Overview: "Mathematics and Probability Theory" (August 25, 2025)**

   The Relevance of Mathematics and Probability Theory for Modern Research in Political Science

Required Reading:

- No specific required readings for the first class, but I will circulate some research articles that use statistical research methods to underscore the importance of mathematics to modern political research. Please pick one that is of greatest interest to you and read it more carefully.

2. **Building Blocks, Part I (September 1, 2025)**

   Preliminaries + Algebra Review

Required Readings:

- **Moore & Siegel,** chap. 1 ("Preliminaries")
- **Moore & Siegel,** chap. 2 ("Algebra Review")

3. **Building Blocks, Part II (September 5, 2025, 11 AM to 12 PM, 1 PM to 2 PM, 3 PM to 4 PM)**

   Functions, Relations, & Utility + Limits and Continuity, Sequences & Series, More On Sets

Required Readings:

- **Moore & Siegel,** chap. 3 ("Functions, Relations, and Utility")
- **Moore & Siegel,** chap. 4 ("Limits and Continuity, Sequences and Series, and More On Sets")

Problem Set 1 Available in This Week!

— NO CLASS ON SEPTEMBER 8, 2025 (HOLIDAY BREAK)! —

— NO CLASS ON SEPTEMBER 15, 2025 (HOLIDAY BREAK)! —

PART II: CALCULUS IN ONE DIMENSION

The second part of the class deals with calculus in one dimension. Specifically, this part covers (1) introduction to calculus, (2) the derivative, (3) rules for differentiation, (4) derivatives of functions, (5) the definite integral, (6) indefinite integrals and the fundamental theorem of calculus, (7) computing integrals, (8) rules of integration, (9) extrema, (10) higher-order derivatives, concavity, and convexity, and (11) finding extrema.

4. Calculus, Part I: Fundamentals (Differentiation) (September 19, 2025, 2 PM)

Introduction to Calculus and the Derivative + The Rules of Differentiation

Required Readings:

- **Moore & Siegel,** chap. 5 ("Introduction to Calculus and the Derivative")
- **Moore & Siegel,** chap. 6 ("The Rules of Differentiation")

Problem Set 1 Due in This Week!

5. Calculus, Part II: The Integral (September 22, 2025)

Required Reading:

- **Moore & Siegel,** chap. 7 ("The Integral")

6. Calculus, Part III: Extrema in One Dimension (September 29, 2025)

Required Reading:

- **Moore & Siegel,** chap. 8 ("Extrema in One Dimension")

Problem Set 2 Available in This Week!

PART III: PROBABILITY THEORY

The third part of the class deals with probability theory. Specifically, it covers (1) basic probability theory, (2) computing probabilities, (3) specific measures of probabilities, (4) the distribution of a single variable, (5) sample distributions, (6) empirical joint and marginal distributions, (7) the probability mass function, (8) the cumulative distribution function, (9) probability distributions and statistical modeling, (10) expectations of random variables, (11) continuous random variables, (12) expectations of continuous random variables, (13) important continuous distributions for statistical modeling.

7. Probability, Part I: An Introduction to Probability **(October 6, 2025)**

Required Reading:

- **Moore & Siegel,** chap. 9 ("An Introduction to Probability")

— NO CLASS ON OCTOBER 13, 2025 (HOLIDAY BREAK)! —

8. Probability, Part II: Discrete Distributions **(October 20, 2025)**

Required Reading:

- **Moore & Siegel,** chap. 10 ("Discrete Distributions")

Problem Set 2 Due in This Week!

9. Probability, Part III: Continuous Distributions **(October 31, 2025, 2 PM)**

Required Reading:

- **Moore & Siegel,** chap. 11 ("Continuous Distributions")

Problem Set 3 Available in This Week!

Recommended Reading (as a general overview of key probability theory insights):

- **Cunningham,** chap. 2, 2.1 (Basic probability theory) to 2.10 (Covariance)

PART IV: LINEAR ALGEBRA

The fourth part of the class deals with linear algebra. Specifically, it covers (1) scalars and vectors, (2) matrices, (3) properties of vectors and matrices, (4) matrix illustration of OLS estimation, (5) vector spaces, (6) solving systems of equations, (7) Eigenvalues, Eigenvectors, and matrix decomposition.

10. Introduction to Linear Algebra (November 3, 2025)

Required Readings:

- **Moore & Siegel,** chap. 12 ("Fun with Vectors and Matrices")
- **Moore & Siegel,** chap. 13 ("Vector Spaces and Systems of Equations")

11. Vector Spaces, Systems of Equations, and Eigenvalues (November 10, 2025)

Required Readings:

- **Moore & Siegel,** chap. 14 ("Eigenvalues and Markov Chains")
- **Moore & Siegel,** chap. 12
  - 12.5 Matrix Illustration of OLS Estimation
  - Errata (important): https://sites.duke.edu/daveasiegel/files/2020/07/Errata.pdf
- **Unknown Author: OLS in Matrix Form (Available here)**

Problem Set 3 Due in This Week!

Problem Set 4 Available in This Week!

PART V: MULTIVARIATE CALCULUS

The fifth part of the class deals with multivariate calculus. Specifically, it covers (1) functions of several variables, (2) calculus in several dimensions, (3) concavity and convexity redux, (4) unconstrained optimization, (5) constrained optimization with equality constraints, (6) constrained optimization with inequality constraints, (7) properties of the maximum and minimum, and (8) implicit differentiation.

12. Introduction to Multivariate Calculus and Multivariate Optimization (November 17, 2025)

Required Readings:

- **Moore & Siegel,** chap. 15 ("Multivariate Calculus")
- **Moore & Siegel,** chap. 16 ("Multivariate Optimization")

PART VI: PRACTICAL APPLICATIONS

The sixth and final part of the class deals with practical applications of the mathematical formulas and insights that we have learned about. In this respect, we will learn how mathematics and probability theory can be used to better understand regression analysis and maximum likelihood estimation.

13. Regression Analysis, Part I: OLS Regression (November 28, 2025, 3 PM)

Required Readings:

- **Gailmard**, chap. 2
  - 2.3.5 Regression
  - 2.3.6 Multiple Regression
  - (2.3.7. Specifying Regression Models) (optional)
- **Pishro-Nik**, chap. 8
  - 8.5 Linear Regression
- **Cunningham,** chap. 2
  - 2.11 Population model
  - 2.12 Mean independence
  - 2.13 Ordinary least squares
  - 2.14 Algebraic Properties of OLS
  - 2.15 Goodness-of-fit
  - 2.16 Expected value of OLS
  - 2.18 CEF decomposition property
  - 2.19 CEF prediction property
  - 2.25 Variance of the OLS estimators
- **Gailmard**, chap. 7
  - 7.5 Sample Regression Coefficients with IID Draws
  - 7.6 Derived Distributions: Sampling from Normal DGPs when Sigma-Squared Must Be Estimated
- **Gailmard**, chap. 8
  - 8.5 Tests about Regression Coefficients

Optional Reading:

- **Gailmard**, chap. 6
  - 6.7 Specifiying Linear Models

Problem Set 4 Due in This Week!

– NO CLASS ON DECEMBER 1, 2025 (HOLIDAY BREAK)! –

14. Regression Analysis, Part II: Generalized Linear Models (December 11, 2025, 2 PM)

(Focus on Binomial and Poisson Regression Models)

Required Readings:

- **Gailmard**, chap. 6
  - 6.2 The Bernoulli and Binomial Distributions: Binary Events
  - 6.3 The Poisson Distribution: Event Counts
- **Gailmard**, chap. 9
  - 9.3 Maximum Likelihood Estimation
- **Pishro-Nik**, chap. 8
  - 8.2.3 Maximum Likelihood Estimation (Also available here)
  - 8.2.4 Asymptotic Properties of MLEs (Also available here)
- **Pawitan**, chap. 4
  - 4.1 Binomial or Bernoulli Models
  - 4.4 Poisson Model
- **Pawitan**, chap. 6
  - 6.2 Logistic Regression Models
  - 6.3 Poisson Regression Models
- **Pishro-Nik**, chap. 8
  - 8.4.5 Likelihood Ratio Tests

Optional Reading:

- **Pishro-Nik**, chap. 11
  - 11.1 Poisson Processes

Dates and Deadlines at the End of the Semester:

- Final Examination: Friday, January 23, 2026, 2.00-5.00 **PM** (14.00-17.00)